AER Benchmark Solution Sheet

1. Test ID: AER-FCM-101

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Date: 02.06.2005

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3. Code or Program Applied: DYN3D Version 3.2

4. Short description of the Code:

The code DYN3D/3.2 applies a 3-dimensional neutron kinetic model and the thermal-hydraulic model FLOCAL [1]. The programming language is FORTRAN90.

The neutron kinetics is calculated by using nodal expansion methods (NEM) for hexagonal or Cartesian geometry. The developed method solves the neutron diffusion equation for two energy groups. Steady states and transient behaviour can be calculated. The code FLOCAL consists of a two-phase coolant flow model, a fuel rod model and a heat transfer regime map up to superheated steam. One coolant channel per fuel assembly and additional hot channels can be considered. Different libraries of neutron diffusion constants are linked with the code.

5. Known Approximations:

- Neutron Kinetics

  - Neutron diffusion theory
  - Two group theory
  - Nodewise homogenized cross sections

- Thermal Hydraulics

  - One-dimensional four equation model for two-phase coolant flow (momentum equation of mixture, energy equation of mixture, mass balance of mixture and mass balance of vapour phase)
  - Constitutive laws
  - Radial heat conduction equation in fuel pin
  - Map for heat transfer from fuel to coolant

- Feedback

  - Calculation of neutron cross section by using libraries or input data

6. Mathematical Model:

- Neutron Kinetics

The 3-dimensional neutron kinetic model is based on the solution of the 3-dimensional 2-group neutron diffusion equation by a nodal expansion method which is specific for the geometry of fuel assemblies. It is assumed that the macroscopic cross sections are spatially constant in a node being a part of the fuel assembly in an axial layer. The diffusion equations in the nodes are solved by transverse integration. Considering the Cartesian geometry, one-dimensional equations for the directions *x,y,z* are obtained by transverse integration over two directions. Considering the hexagonal-z geometry, the transverse integration is used for splitting the 3-dimensional problem into a 2-dimensional problem in the hexagonal plane and a 1-dimensional problem in axial direction. In two methods, called HEXNEM1 and HEXNEM2 for the hexagonal plane, the flux of each energy group is expanded into two-dimensional polynomials up to the second order and into 6 (HEXNEM1) or 12 (HEXNEM2) exponential functions being the solutions of the homogeneous Helmholtz equation [2]. A similar flux expansion is used for the one-dimensional problems of the axial direction and for the Cartesian geometry [3]. Leakage, fission and scattering terms are approximated by the polynomials. Considering the hexagonal plane, the nodes are coupled only by the side-averaged fluxes and currents in the HEXNEM1 method. Additionally, the nodes are coupled by the corner values of fluxes and currents in the HEXNEM2 method. The HEXNEM2 method is more accurate in comparison to HEXNEM1 for reactors with larger assembly diameters like the VVER-1000 [4].

The steady state diffusion equation is solved by an inner and outer iteration process. The outer iterations are the fission source iterations accelerated by a Chebychev extrapolation scheme. A small number of inner iterations (3 - 5) are sufficient for the convergence.

Optionally, assembly discontinuity factors can be considered at the boundaries of the assemblies.

Concerning the time integration over the neutronic time step an implicite difference scheme with exponential transformation is used. The exponents in each node are calculated from the previous time step or during the iteration process. For the calculation of matrix elements describing the relation between partial currents and averaged fluxes it is assumed that the time behaviour of the neutron fluxes in the nodes is exponential and the local variation of the source of delayed neutrons is proportional to the source of prompt neutrons. These assumptions allow the same treatment of diffusion equation in the nodes as in the steady state. In the iteration process an inhomogeneous problem has to be solved. Similar methods as used for the steady state are applied.

During the transient, the individual reactivity contributions can be calculated with the help of the actual flux distribution and the adjoint fluxes of the initial steady state.

- Thermal Hydraulics

The thermal-hydraulic model of the reactor core and the fuel rod model are implemented in the module FLOCAL [5] being a part of DYN3D. The reactor core is modelled by parallel

cooling channels which can describe one or more fuel assemblies. The parallel channels are coupled hydraulically by the condition of equal pressure drop over all core channels. Additionally, so- called hot channels can be considered for the investigation of hot spots and uncertainties in power density, coolant temperature or mass flow rate. Thermal-hydraulic boundary conditions for the core like coolant inlet temperature, pressure, coolant mass flow rate or pressure drop must be given as input for DYN3D. DYN3D and the system code ATHLET (GRS) are linked by internal, external and parallel coupling.

7. Features of Techniques Used:

Regarding the specification, steady-state neutronic calculations were performed.

8. Computer, Operational System: PC with Linux or Windows, SUN workstation with Solaris

9. References:

[1] U. Grundmann, U. Rohde, and S. Mittag "DYN3D – Three Dimensional Core Model for Steady-State and Transient Analysis of Thermal Reactors", Proceedings of the 2000 ANS International Topical Meeting on Advances in Reactor Physics and Mathematics and Computation into the Next Millennium (PHYSOR 2000), Pittsburgh (USA), May, 7 – 11, 2000.

[2] Grundmann U., Hollstein F., A Two-Dimensional Intranodal Flux Expansion Method for Hexagonal Geometry, Nucl. Sci Engeng, 133, (1999) 201- 212.

[3] U. Grundmann, U.; Rohde, U.; "DYN3D - A 3-dimensional Core Model for Steady State and Transient Analysis in Thermal Reactors", Proceedings of the International Confer-ence on the Physics of Reactors "PHYSOR 96", pp. J70 – J79, Mito(Japan), September, 16-20, 1996.

[4] Grundmann, "HEXNEM – A Nodal Method for the Solution of the Neutron Diffusion Equation in Hexagonal Geometry", Proceedings of the M&C'99- Conference on Mathematics and Computations in Nuclear Applications, pp. 1086-1095, Madrid, September, 27 – 30, 1999.

[5] U. Rohde, "Modelling of Fuel Rod Behaviour and Heat Transfer in the Code FLOCAL for Reactivity Accident Analysis of Reactor Cores", 1st Baltic Heat Transfer Conference, Göteborg, (1991), published in: Transport Processes in Engineering, 2: Elsevier Publ., Amsterdam, (1992).

[6] N.P.Kolev, R.Lenain, C.Fedon-Magnaud, "CRONOS Solutions of the AER 3D Benchmark for VVER-1000", CEA Internal Report, Saclay, 1997.

[7] N.P.Kolev, R.Lenain, C.Fedon-Magnaud, "AER-FCM-101 Benchmark Specification Sheet", AER Benchmark Book, AEKI-KFKI (Hungary).

10. Results:

The DYN3D calculations of the AER FCM-101 benchmark [7] were performed with HEXNEM1 and HEXNEM2 by using 10 core layers and 1 node/assembly in each layer.

a) *HEXNEM1*

Table 1: Eigenvalue k_{eff}, 3D normalized powers $P_{i,j}$ and assembly powers P_i^{ass}.

k_{eff} = 1.04994

ass i	$P_{i,j}$ axial layer j										P_i^{ass}
	1	2	3	4	5	6	7	8	9	10	
1	0.835	1.813	2.259	1.771	1.456	1.087	0.741	0.492	0.379	0.163	1.100
2	0.612	1.333	1.715	1.661	1.400	1.034	0.696	0.451	0.281	0.117	0.930
3	0.627	1.374	1.817	1.875	1.591	1.126	0.733	0.464	0.271	0.109	0.999
4	0.607	1.328	1.747	1.788	1.519	1.092	0.719	0.458	0.269	0.109	0.964
5	0.830	1.826	2.441	2.557	2.141	1.367	0.855	0.537	0.308	0.123	1.299
6	0.660	1.452	1.936	2.021	1.702	1.134	0.719	0.452	0.260	0.104	1.044
7	0.656	1.447	1.946	2.055	1.743	1.141	0.730	0.463	0.266	0.105	1.055
8	0.665	1.466	1.967	2.071	1.721	0.907	0.562	0.353	0.202	0.080	0.999
9	0.825	1.817	2.435	2.562	2.155	1.385	0.872	0.549	0.315	0.125	1.304
10	0.736	1.624	2.186	2.326	2.042	1.508	1.019	0.655	0.378	0.149	1.262
11	0.627	1.383	1.861	1.976	1.711	1.202	0.796	0.509	0.293	0.116	1.047
12	0.627	1.383	1.862	1.972	1.699	1.176	0.772	0.492	0.283	0.112	1.038
13	0.405	0.894	1.203	1.285	1.143	0.871	0.601	0.388	0.224	0.089	0.710
14	0.607	1.340	1.804	1.925	1.707	1.290	0.886	0.571	0.330	0.131	1.059
15	0.669	1.476	1.989	2.121	1.877	1.412	0.967	0.623	0.360	0.143	1.164
16	0.545	1.204	1.623	1.730	1.530	1.149	0.786	0.506	0.292	0.116	0.948
19	0.355	0.785	1.057	1.130	1.011	0.778	0.541	0.351	0.203	0.081	0.629
20	0.438	0.967	1.302	1.393	1.247	0.960	0.669	0.433	0.251	0.100	0.776

b) *HEXNEM2*

Table 2: Eigenvalue k_{eff}, 3D normalized powers $P_{i,j}$ and assembly powers P_i^{ass}.

k_{eff} = 1.04940

ass i	$P_{i,j}$ axial layer j										P_i^{ass}
	1	2	3	4	5	6	7	8	9	10	
1	0.856	1.858	2.316	1.821	1.495	1.114	0.756	0.500	0.382	0.164	1.126
2	0.630	1.370	1.762	1.704	1.434	1.056	0.708	0.457	0.283	0.118	0.952
3	0.643	1.408	1.861	1.916	1.622	1.144	0.741	0.468	0.272	0.109	1.018
4	0.625	1.365	1.795	1.833	1.554	1.114	0.730	0.463	0.271	0.110	0.986
5	0.846	1.860	2.483	2.596	2.168	1.378	0.857	0.536	0.306	0.122	1.315
6	0.675	1.483	1.976	2.059	1.729	1.148	0.725	0.454	0.260	0.103	1.061
7	0.665	1.467	1.969	2.076	1.755	1.141	0.725	0.457	0.261	0.103	1.062
8	0.677	1.491	1.998	2.100	1.741	0.916	0.564	0.353	0.201	0.080	1.012
9	0.838	1.845	2.471	2.595	2.176	1.391	0.870	0.546	0.311	0.123	1.317
10	0.734	1.620	2.178	2.313	2.022	1.482	0.994	0.634	0.364	0.144	1.249
11	0.632	1.394	1.875	1.985	1.713	1.195	0.786	0.500	0.286	0.113	1.048
12	0.634	1.399	1.880	1.988	1.706	1.174	0.765	0.485	0.277	0.109	1.042
13	0.398	0.879	1.181	1.258	1.115	0.844	0.578	0.371	0.214	0.084	0.692
14	0.602	1.328	1.786	1.901	1.679	1.261	0.859	0.551	0.317	0.125	1.041
15	0.666	1.471	1.979	2.105	1.856	1.386	0.942	0.604	0.347	0.137	1.149
16	0.547	1.206	1.624	1.727	1.522	1.136	0.771	0.494	0.284	0.112	0.942
19	0.348	0.769	1.034	1.103	0.983	0.752	0.520	0.335	0.193	0.076	0.611
20	0.432	0.953	1.282	1.368	1.220	0.934	0.646	0.416	0.240	0.095	0.758

11. Comparison to Recommended Solution:

The comparisons were performed with the recommended reference solution of table 2 of [6,7].

a) HEXNEM1:

Table 3: Deviations of eigenvalue k_{eff}, 3D normalized powers $P_{i,j}$.

$\Delta k_{eff} = 41$ pcm

ass i	$(P_{i,j} - P_{i,j,ref}) \cdot 100$									
	axial layer j									
	1	2	3	4	5	6	7	8	9	10
1	-1.95	-2.81	**-4.03**	-3.42	-2.88	-2.06	-1.44	-0.86	-0.64	-0.36
2	-1.50	-2.52	-3.14	-3.25	-2.63	-1.81	-1.30	-0.80	-0.43	-0.21
3	-1.39	-2.31	-2.85	-2.80	-2.21	-1.48	-0.97	-0.56	-0.34	-0.12
4	-1.59	-2.73	-3.36	-3.32	-2.71	-1.95	-1.28	-0.84	-0.49	-0.21
5	-1.29	-1.86	-2.51	-2.36	-1.84	-1.53	-0.83	-0.38	-0.18	-0.15
6	-1.25	-1.95	-2.49	-2.40	-1.85	-1.37	-0.87	-0.45	-0.26	-0.13
7	-0.65	-0.85	-1.14	-1.07	-0.77	-0.52	-0.23	0.09	0.05	0.02
8	-0.98	-1.41	-1.83	-1.81	-1.79	-0.93	-0.45	-0.28	-0.17	-0.06
9	-1.13	-1.42	-2.01	-1.90	-1.32	-1.12	-0.55	-0.17	-0.03	-0.02
10	0.36	1.60	1.70	1.91	1.91	1.81	1.32	1.05	0.76	0.25
11	-0.35	-0.25	-0.46	-0.43	-0.11	0.07	0.14	0.24	0.21	0.11
12	-0.51	-0.59	-0.84	-0.85	-0.45	-0.17	-0.01	0.13	0.19	0.00
13	0.60	1.63	1.84	1.98	1.91	1.49	1.08	0.79	0.64	0.20
14	0.49	1.92	2.11	**2.26**	2.17	1.71	1.28	0.93	0.73	0.29
15	0.36	1.64	1.77	1.97	2.00	1.77	1.26	1.02	0.80	0.26
16	0.03	0.50	0.61	0.61	0.69	0.70	0.56	0.42	0.41	0.17
19	0.64	1.79	1.98	2.12	2.00	1.62	1.25	0.88	0.62	0.26
20	0.57	1.68	1.93	2.10	1.96	1.64	1.18	0.85	0.70	0.25

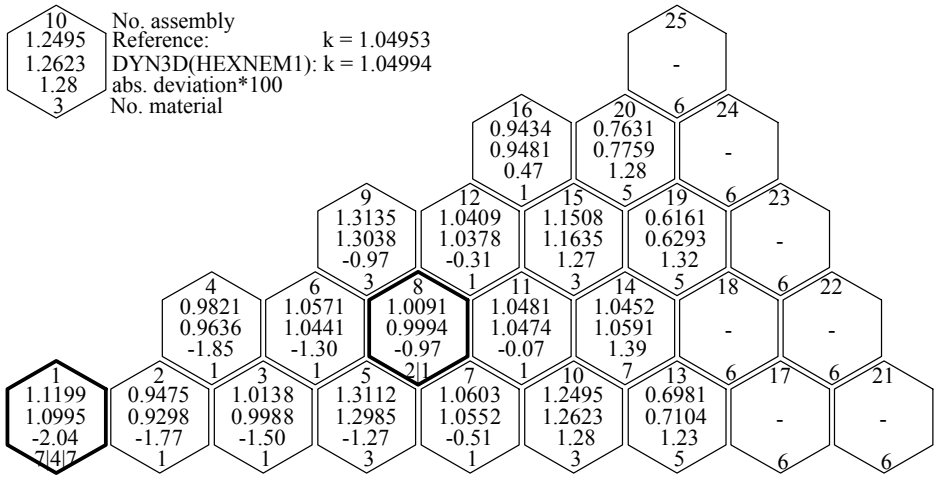

Fig. 1: HEXNEM1 - Absolute deviations of assembly powers P_i^{ass}

b) HEXNEM2:

Table 4: Deviations of eigenvalue k_{eff}, 3D normalized powers $P_{i,j}$.

Δk_{eff} = -13 pcm

ass i	$(P_{i,j} - P_{i,j,ref}) \cdot 100$									
	axial layer j									
	1	2	3	4	5	6	7	8	9	10
1	0.17	**1.73**	1.66	1.55	1.05	0.56	0.08	-0.07	-0.31	-0.27
2	0.26	1.25	1.57	1.09	0.75	0.38	-0.08	-0.19	-0.15	-0.12
3	0.21	1.11	1.47	1.35	0.92	0.27	-0.16	-0.25	-0.24	-0.10
4	0.17	1.05	1.42	1.20	0.77	0.21	-0.15	-0.31	-0.26	-0.13
5	0.27	1.46	1.65	1.54	0.86	-0.40	-0.57	-0.47	-0.35	-0.24
6	0.23	1.22	1.50	1.40	0.92	0.00	-0.35	-0.34	-0.29	-0.16
7	0.25	1.05	1.18	0.96	0.37	-0.49	-0.71	-0.49	-0.40	-0.18
8	0.20	1.10	1.30	1.09	0.19	-0.02	-0.19	-0.31	-0.27	-0.12
9	0.22	1.45	1.56	1.36	0.79	-0.50	-0.68	-0.54	-0.37	-0.18
10	0.24	1.24	0.94	0.58	-0.09	-0.79	-1.19	-0.96	-0.57	-0.31
11	0.20	0.89	0.85	0.53	0.07	-0.62	-0.89	-0.73	-0.48	-0.19
12	0.24	0.98	1.04	0.74	0.30	-0.44	-0.73	-0.63	-0.37	-0.25
13	-0.06	0.10	-0.36	-0.66	-0.88	-1.17	-1.17	-0.90	-0.43	-0.25
14	-0.02	0.71	0.25	-0.14	-0.63	-1.20	**-1.35**	-1.11	-0.60	-0.27
15	0.14	1.08	0.77	0.42	-0.11	-0.76	-1.16	-0.91	-0.48	-0.28
16	0.16	0.73	0.71	0.33	-0.14	-0.61	-0.88	-0.81	-0.43	-0.20
19	-0.08	0.17	-0.34	-0.61	-0.83	-1.02	-0.95	-0.75	-0.42	-0.17
20	-0.04	0.28	-0.11	-0.39	-0.74	-1.01	-1.11	-0.90	-0.42	-0.22

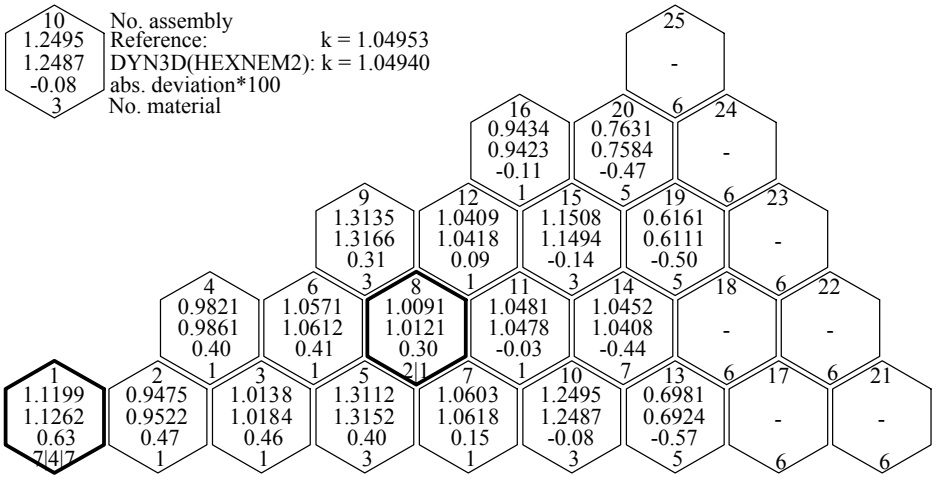

Fig. 2: HEXNEM2 - Absolute deviations of assembly powers P_i^{ass}

c) Overview

Table 5: Deviation of k_{eff}, maximum and averaged deviations of node and assembly powers for HEXNEM1 and HEXNEM2.

Method	$\left\|\Delta k_{eff}\right\|$ (pcm)	$100 \cdot \max_i \left\|\Delta P_i\right\|$	$\frac{100}{N}\sum_{i=1}^{N}\left\|\Delta P_i\right\|$	$100 \cdot \max_i \left\|\Delta P_i^{ass}\right\|$	$\frac{100}{N_{ass}}\sum_{i=1}^{N_{ass}}\left\|\Delta P_i^{ass}\right\|$
HEXNEM1	41	4.03	1.09	2.04	1.07
HEXNEM2	13	1.73	0.59	0.63	0.31